

Question Paper
Quantitative Methods-I (131) : October 2004

- • Answer all questions.
- • Marks are indicated against each question.

1. The number of ways in which all the letters of the word "INTEGRATION" can be arranged so that all vowels are always in the beginning of the word is ≤
Answer
≥

(a) 10800 (b) 60480 (c) 120960 (d) 1260 (e) 720.

(1 mark)

2. Find the sum of n terms of the series:

$$\log a + \log \frac{a^2}{b} + \log \frac{a^3}{b^2} + \log \frac{a^4}{b^3} + \dots \text{ to } n \text{ terms.}$$

- (a) $(n/2) [n \log ab - \log (a/b)]$ (b) $(n/2) [\log ab - n \log (a/b)]$
 (c) $(n/2) [n \log ab + \log (a/b)]$ (d) $(n/2) [\log ab + n \log (a/b)]$
 (e) $(n/2) [\log (a/b) - n \log ab].$

(2 marks)

3. The first and last term of an A.P. are a and l respectively. If S is the sum of all the terms of the A.P., then the common difference of the A.P. is ≤
Answer
≥

- (a) $\frac{l^2 - a^2}{2S - (l + a)}$ (b) $\frac{l^2 + a^2}{2S - (l + a)}$ (c) $\frac{l^2 - a^2}{2S + l + a}$ (d) $\frac{l^2 + a^2}{2S + l + a}$ (e) $\frac{l^2 - a^2}{(l + a) - 2S}$

(1 mark)

4. $x = \frac{m^{\frac{m-1}{n}}}{n^{\frac{n+1}{m}}}$, $y = \frac{n^{\frac{1-n}{m}}}{m^{\frac{m+3}{n}}}$ and $z = \frac{m^{\frac{5}{n}}}{n^{\frac{1-2n}{m}}}$ ≤
Answer
≥

The value of xyz is

- (a) $\frac{\sqrt[n]{m}}{\sqrt[n]{n}}$ (b) $\frac{\sqrt[n]{m}}{\sqrt[n]{n}}$ (c) $\frac{\sqrt[n]{m}}{\sqrt[n]{m}}$ (d) $\frac{\sqrt[n]{n}}{\sqrt[n]{m}}$ (e) $\sqrt[n]{m} \times \sqrt[n]{n}$

(1 mark)

5. $2x - 3y + 3z = 7$
 $y - x + z = 5$
 $2x - 2y + z = 2$

Answer
 $\leq$
 $\geq$

The value of x in the above simultaneous equations would be

- (a) 1 (b) 2 (c) 3 (d) 4 (e) 5.

(1 mark)

6. The sum of four numbers of a G.P is 85 and the product of the same is 4096. The highest of the three numbers is

Answer
 $\leq$
 $\geq$

- (a) 56 (b) 64 (c) 81 (d) 128 (e) 144.

(2 marks)

7. If ${}^{28}C_{2r} : {}^{24}C_{2r-4} = 225 : 11$. The value of r is

Answer
 $\leq$
 $\geq$

- (a) 9 (b) 8 (c) 7 (d) 6 (e) 5.

(2 marks)

8. Two dice are thrown simultaneously. The probability that the sum of the numbers on them is less than or equal to 5 is

Answer
 $\leq$
 $\geq$

- (a) $1/9$ (b) $1/6$ (c) $2/9$ (d) $5/18$ (e) $1/3$.

(1 mark)

9. If $f(x) = \log \frac{1+x}{1-x}$, then $f\left(\frac{2x}{1+x^2}\right) =$

Answer
 $\leq$
 $\geq$

- (a) 0 (b) 1 (c) $f(x)$ (d) $2f(x)$ (e) $3f(x)$.

(1 mark)

10. If $x = \frac{1+t}{t^3}$ and $y = \frac{3+4t}{2t^2}$ then $x(y')^3 =$

Answer
 $\leq$
 $\geq$

($y' = dy/dx$ in the given expression)

- (a) $1 - y'$ (b) $1 + y'$ (c) $y' - 1$ (d) y' (e) 1.

(2 marks)

11. A class consists of a number of boys whose ages are in A.P. with a common difference of four months. If the youngest boy is just eight years old and if the sum of the ages is 168 years, find the number of boys.

Answer
 $\leq$
 $\geq$

- (a) 12 (b) 14 (c) 16 (d) 18 (e) 20.

(1 mark)

12. Find the sum of all natural numbers between 250 and 1000 which are

$\leq$

exactly divisible by 3.

- (a) 156250 (b) 156500 (c) 312750 (d) 156000 (e) 156375.

Answer
≤
≥

(2 marks)

13. Which of the following intervals, defined by the end points **a** and **b**, excludes the real numbers which are less than or equal to **a**, or, greater than or equal to **b**?

- (a) (a, b) (b) [a, b] (c) (a, b] (d) [a, b) (e) {a, b}.

Answer
≤
≥

(1 mark)

14. A guard of 12 persons is to be formed from a group of **n** soldiers in all possible ways. If the number of times two particular soldiers are working together is **2m** and the number of times three particular soldiers are working together is **m**, the value of **n** is

- (a) 18 (b) 20 (c) 22 (d) 24 (e) 26.

Answer
≤
≥

(1 mark)

15. If $\frac{\log a}{(b-c)} = \frac{\log b}{(c-a)} = \frac{\log c}{(a-b)}$; the value of $a^a b^b c^c$ would be

- (a) a (b) b (c) c (d) 1 (e) 0.

Answer
≤
≥

(1 mark)

16. If $u = \frac{x+y}{x-y}$ then $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} =$

- (a) $\frac{2}{x+y}$ (b) $\frac{1}{x+y}$ (c) $\frac{2}{x-y}$ (d) $\frac{1}{x-y}$ (e) $\frac{x-y}{x+y}$.

Answer
≤
≥

(1 mark)

17. If $y^2 = (x-b)(x-c)$, then $\frac{d^2 y}{dx^2}$ at $x = b+c$, is

- (a) $\frac{1}{(bc)^{3/2}}$ (b) $\frac{1}{4(bc)^{3/2}}$ (c) $\frac{bc-b^2}{4(bc)^{3/2}}$ (d) $\frac{-(b-c)^2}{4(bc)^{3/2}}$
(e) $\frac{(b-c)^2}{(bc)^{3/2}}$.

Answer
≤
≥

(2 marks)

18. If $x^{\log_x(x^2-4x+5)} = (x-1)$ then $x =$ ≤
Answer
≥

(a) 0 (b) 1 (c) 2 (d) 4 (e) 5. ≥

(1 mark)

19. If $e^{f(x)} = \frac{1+x}{1-x}$, $f(x) = kf\left(\frac{2x}{1+x^2}\right)$, then $k =$ ≤
Answer
≥

(a) 1 (b) 2 (c) 3 (d) 1/3 (e) 1/2. ≥

(1 mark)

20. If $f(x) = \sqrt{\frac{(x+1)(x-2)}{(x+3)(x+6)}}$, then $\frac{f'(0)}{f(0)} =$ ≤
Answer
≥

(a) 1/18 (b) 1/9 (c) 1/3 (d) 1 (e) 0. ≥

(1 mark)

21. The p^{th} term of a H.P. is qr and q^{th} term is rp . The r^{th} term of the H.P. is ≤
Answer
≥

(a) $\frac{p}{q}$ (b) pq (c) $\frac{q}{p}$ (d) $\frac{rp}{q}$ (e) $\frac{rq}{p}$. ≥

(1 mark)

22. If $\log_{12} 27 = a$, the value of $\log_6 16$ would be ≤
Answer
≥

(a) $\frac{4(a-3)}{(a+3)}$ (b) $\frac{(3-a)}{4(3+a)}$ (c) $\frac{4(3+a)}{(3-a)}$ (d) $\frac{4(3-a)}{(3+a)}$ (e) $\frac{(3-a)}{(3+a)}$ ≥

(2 marks)

23. $\lim_{t \rightarrow 0} \frac{2 - \sqrt{4-t^2}}{t^2} =$ ≤
Answer
≥

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) 4 (e) $\sqrt{2}$ ≥

(1 mark)

24. $f(x) = \sqrt{\log\left[\frac{(5x-x^2)}{6}\right]}$ ≤
Answer
≥

The domain of the above function is ≥

(a) (2, 3) (b) [2, 3) (c) [2, 3] (d) (2, 3] (e) (2, 4) ≥

(1 mark)

25. Differentiation of a^{bx-c} with respect to x is ≤
Answer
≥

(a) $(bx - c)a^{bx-c-1} \times \log a$

(b) $(bx - c)a^{bx-c-1}$

≤

(c) $ba^{bx-c} \times \log a$

(d) $\frac{(bx - c)a^{bx-c-1}}{\log a}$

(e) $\frac{ba^{bx-c}}{\log a}$

(1 mark)

26. If $\sqrt{x} + \sqrt{y} = 4$ then $\frac{dy}{dx} =$

(a) $-\sqrt{\frac{y}{x}}$

(b) $-\sqrt{\frac{x}{y}}$

(c) $\sqrt{\frac{y}{x}}$

(d) $\sqrt{\frac{x}{y}}$

(e) $\sqrt{\frac{x}{x+y}}$

(1 mark)

27. $\int_0^1 2xe^{x^2+5} dx =$

(a) e

(b) e^6

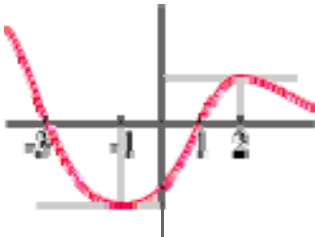
(c) $e^6(e - 1)$

(d) e^5

(e) $e^5(e - 1)$

(1 mark)

28.



The function f whose graph is shown above

(a) Increases monotonically in the interval $[-3, -1]$

(b) Decreases monotonically in the interval $[-1, 2]$

(c) Is positive in the open interval $(-3, 1)$

(d) Satisfies $f(2) > f(0)$

(e) Satisfies $f(1) > f(2)$.

(1 mark)

29. $f(x) = 50x^2 - 400x + 1000$

$f(x)$ is minimum at $x =$

(a) 1

(b) 4

(c) 40

(d) 100

(e) 400.

(1 mark)

30. The total cost of production in a day, for a firm, is given by the following equation:

≤
Answer
≥

$$T(x) = 10500x - 20x^2 + \frac{1}{5}x^3$$

Where $T(x)$ = Total cost of production in a day
 x = Number of units produced in a day

How many units of the product should be produced in a day so that the average cost of the product is minimum?

- (a) 5 (b) 10 (c) 20 (d) 50 (e) 100.

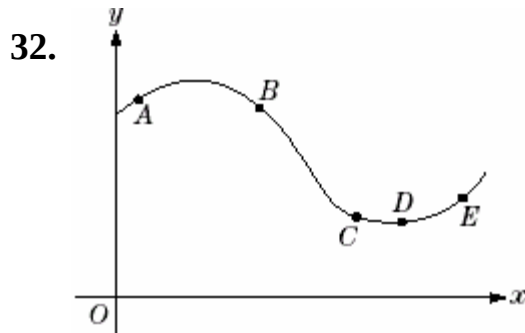
(2 marks)

31. The demand and cost functions for a product are given by $P = 1200 - 6Q$,
 $C = Q^3 - 6Q^2 + 120Q + 600$.

The marginal cost and marginal revenue at an output of 15 units of the product are

- (a) 650 and 1250 respectively (b) 615 and 1200 respectively
 (c) 600 and 1120 respectively (d) 615 and 1020 respectively
 (e) 600 and 1200 respectively.

(1 mark)



At which of the five points on the graph both $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ are negative?

- (a) A (b) B (c) C (d) D (e) E

(1 mark)

33. Royce Electric Co., a manufacturer of gas dryers, produces two models: a standard (STD) model and a deluxe (DEL) model. Production consists of two major phases. In the first phase, stamping and painting (S & P), sheet metal is formed (stamped) into the appropriate components and painted. In the second phase, assembly and testing (A & T), the sheet metal components along with the motor and controls are assembled and tested. (Ignore any scheduling problems that might arise from the sequential nature of the operations.) Information concerning the resource

requirements and availability is shown in the following table:

Quantity of Resources Required per Unit of Output

Resource	Dryer Type		Quantity of Resources Available During the Period
	<i>STD</i>	<i>DEL</i>	
<i>S & P</i> (hours)	1.0	2.0	2,000
Motors (units)	1.0	1.0	1,400
<i>A & T</i> (hours)	1/3	1.0	900
Profit contribution (Rs./unit)	100	125	

The objective is to maximize the total contribution towards profit.

The total number of *STD* dryers to be produced is

- (a) 800 (b) 750 (c) 700 (d) 650 (e) 600.

(2 marks)

34. The CSTC, Calcutta State Transport Corporation, is considering the purchase of additional buses to expand its service. Two different models are being considered. A small model would cost Rs.10,00,000, carry 45 passengers, and operate at an average speed of 25 kilometers per hour over the existing bus routes. A larger model would cost Rs. 15,00,000, carry 55 passengers, and operate at an average speed of 30 kilometers per hour. The transport authority has Rs. 3,00,00,000 in its *capital* budget for purchasing new buses during the forthcoming year.

However, the authority is also restricted in its expansion program by limitations imposed on its *operating* budget. Specifically, a hiring freeze is in effect and only 25 drivers are available for the foreseeable future to operate any new buses that are purchased. To plan for increased future demand, the transport authority wants at least one-half of all new buses purchased to be the larger model. The transport authority wishes to determine how many buses of each model to buy to maximize additional capacity measured in passenger-kms-per hour while satisfying these constraints. Using the linear-programming framework, let X_1 be the number of small buses purchased and X_2 the number of large buses purchased.

The value of X_2 at optimal solution is

- (a) 15 (b) 14 (c) 13 (d) 12 (e) 11.

(2 marks)

35. If the objective is to maximize $f(x, y) = 143x + 60y$ subject to the following constraints: $\leq$
Answer
 $\geq$

$$\begin{aligned}x + y &\leq 100, \\120x + 210y &\leq 15000, \\110x + 30y &\leq 4000, \\x, y &\geq 0.\end{aligned}$$

The value of y at the optimum solution would be

- (a) 20 (b) 50 (c) 60 (d) 100 (e) 110.

(1 mark)

36. Zaheda has 31 days to complete her quilt for the county fair. The blue squares in the quilt can be sewn at a rate of 4 squares per day, and the white squares at a rate of 7 squares per day. The quilt can have up to 96 squares total. The blue fabric costs about Rs. 0.80 per square and the white fabric costs about Rs. 1.20 per square. Zaheda wants to keep costs at a minimum. Which of the following would be the **true** inequality, which expresses the constraint with regard to the time available to Zaheda? $\leq$
Answer
 $\geq$

[Note: The number of blue and white squares are b and w respectively]

- (a) $\frac{b}{4} + \frac{w}{7} \leq 96$ (b) $\frac{b}{7} + \frac{w}{4} \leq 31$ (c) $\frac{b}{4} + \frac{w}{7} \leq 31$ (d) $\frac{b}{7} + \frac{w}{4} \leq 96$
(e) $4b + 7w \leq 31$

(1 mark)

37. It is required to compute the variance of a population. The observations from the population were grouped into class intervals and, the deviations of the class mid points from an assumed mean were computed. The following details are available: $\leq$
Answer
 $\geq$

Deviation of class mid point from assumed mean	Number of observations
-20	5
-10	10
0	20
-+10	30
20	20
30	15
Total number of observations	100

The width of the class interval is 10. What is the variance of the population?

- (a) 184.75 (b) 1847.5 (c) 18475 (d) 18550 (e) 19450.

(2 marks)

38. A train runs 25 kilometers at a speed of 30km per hour; another 50 kilometers at a speed of 40 km per hour; then it travels for 6 minutes at a speed of 100 km per hour. Lastly, it concluded its journey by moving at a speed of 24 km per hour for a distance of 24 km. What was the average speed?

- (a) 33.28 km per hour (b) 33.78 km per hour
(c) 34.28 km per hour (d) 34.78 km per hour
(e) 35.28 km per hour.

(1 mark)

39. In a textile factory there are 50 skilled workers, 125 semiskilled workers and 75 unskilled workers. It has been observed that on average a unit length of a particular fabric is woven by a skilled worker in 4 hours, by a semiskilled worker in 5 hours and by an unskilled worker in 6 hours. After three years of experience the semiskilled workers are expected to become skilled and the unskilled workers to become semiskilled. It is assumed that there will be no turnover of the workers within the next three years. What will be the change in time taken for weaving an unit length of the same fabric, after three years?

- (a) No change (b) Reduction by 1.60 hour
(c) Reduction by 0.80 hour (d) Increase by 0.80 hour
(e) Increase by 1.60 hours.

(1 mark)

40. An incomplete frequency distribution is given as follows:

Variable	10–	20–	30–	40–50	50–60	60–	70–	Total
	2	3	4			7	8	
	0	0	0			0	0	
Frequency	12	30	f_1	65	f_2	25	18	229

Given the median value is 46, determine the missing frequencies?

- (a) $f_1 = 33$ $f_2 = 46$

- (b) $f_1 = 32$ $f_2 = 47$
- (c) $f_1 = 30$ $f_2 = 49$
- (d) $f_1 = 28$ $f_2 = 51$
- (e) $f_1 = 26$ $f_2 = 53$.

(2 marks)

41. A box contains 20 light bulbs, 5 of which are known to be defective. Three light bulbs are selected at random without replacement. Which of the following is/are correct? ≤
Answer
≥

- (a) The probability that all the three bulbs selected at random are defective

$$= \frac{1}{1140}$$
- (b) The probability that exactly one of the three bulbs selected at random is defective $= \frac{7}{76}$
- (c) The probability that at least one of the three bulbs selected at random is defective $= \frac{137}{228}$
- (d) The probability that at least two of the three bulbs selected at random are not defective $= \frac{91}{228}$
- (e) The probability that at least one of the three bulbs selected at random is not defective $= \frac{113}{114}$

(2 marks)

42. Three candidates, A, B and C, are being considered by the board of directors of a company for the post of the CEO. The probability that A will be appointed as the CEO is 0.20, the probability that B will be appointed is 0.30 and the probability that C will be appointed to the post is 0.50. If A is appointed as the CEO then the probability of successfully launching a new product is 30%. If B is appointed as the CEO then the probability of successfully launching a new product is 50% and if C is appointed then the probability of successfully launching a new product is 60%. ≤
Answer
≥

What is the probability that a new product will be successfully launched?

- (a) 6% (b) 15% (c) 30% (d) 51% (e) 60%.

(2 marks)

43. One shot is fired from each of three guns. G_1 , G_2 and G_3 denote the events that the target is hit by the first, second and third guns respectively. If $P(G_1) = 0.5$, $P(G_2) = 0.6$ and $P(G_3) = 0.8$ and G_1 , G_2 and G_3 are ≤
Answer
≥

independent of each other. The probability of registering exactly one hit will be ≤

- (a) 0.22 (b) 0.24 (c) 0.26 (d) 0.28 (e) 0.30.

(2 marks)

44. The probability that both **S** and **T** occur, the probability that **S** occurs and **T** does not occur while the probability that **T** occurs but **S** does not occur are all equal to **p**. The probability that either **S** or **T** occurs is ≤
Answer
≥

- (a) p (b) $2p$ (c) $3p$ (d) $3p^2$ (e) p^3 .

(2 marks)

45. An NGO has been studying the literacy level among the male and female adults, in a certain village. 3 out of every 5 adults in the village are males. It is observed that 1 out of every 2 adult males, is illiterate and, 4 out of every 5 adult females are illiterates. ≤
Answer
≥

If an adult is randomly selected and found to be illiterate then, what is the probability that the adult is a female?

- (a) 32% (b) 25.8% (c) 51.6% (d) 12.9% (e) 62%.

+++++(2 marks)

46. If **A** and **B** are any two events, the probability that exactly one of them occurs is given by ≤
Answer
≥

- (a) $P(A) + P(B) - 2P(AB)$ (b) $P(A) + P(B) - P(AB)$
(c) $P(\bar{A}) + P(\bar{B}) - 2P(\bar{A}\bar{B})$ (d) $P(\bar{A}) + P(\bar{B}) - P(\bar{A}\bar{B})$
(e) $P(\bar{A}B) + P(A\bar{B})$.

(1 mark)

47. Which of the following is **true** with regard to the classical approach to probability? ≤
Answer
≥

- (a) It assumes that the outcomes are not equally likely
(b) The probability of an event is determined after performing the experiment a large number of times
(c) The probability of an event is determined before performing the experiment
(d) It assumes that all possible outcomes of the experiment are not known
(e) The classical approach cannot be used to find out the probability of mutually exclusive events.

(1 mark)

48. The events **B**, **C** and **D** are mutually exclusive and collectively exhaustive; **A** is another event, which can jointly occur with **B**, **C** or **D**. ≤
Answer
≥

$$P(A \text{ and } B) + P(A \text{ and } C) + P(A \text{ and } D) =$$

- (a) 1.00 (b) $P(D)$ (c) $P(C)$ (d) $P(B)$ (e) $P(A)$.

(1 mark)

49. Consider the series, 1,2,3,4...200. If a number is chosen at random, what is the probability that the chosen number is divisible by 6 or 8? ≤
Answer
≥

- (a) 0.20 (b) 0.25 (c) 0.30 (d) 0.35 (e) 0.40.

(1 mark)

50. A firm manufactures steel pipes in three plants viz, A, B and C. The daily production volumes from the three firms A, B and C respectively are 1000 units, 2000 units and 4000 units respectively. It is known from past experience that 2% of the output from plant A, 3% of the output from plant B and 5% of the output from plant C are defective. A pipe is selected from a days total production and found to be defective. What is the probability that the pipe is manufactured by plant B? ≤
Answer
≥

- (a) 14.29% (b) 21.43% (c) 28.57% (d) 4% (e) 57.14%.

(2 marks)

51. **a**, **b** and **c** are three different real numbers, they are related as $\mathbf{a} < \mathbf{c} < \mathbf{b}$. If the value of **b** is negative which of the following statements would be true? ≤
Answer
≥

for the value of the expression $\left(\frac{\mathbf{b}}{\mathbf{a}} - \frac{\mathbf{c}}{\mathbf{b}} \right)$?

- (a) It is greater than zero (b) It is less than zero
(c) It is greater than one (d) It is less than one
(e) It falls between zero and one.

(1 mark)

52. According to the inverse property of addition ≤
Answer
≥

- (a) For every real number there exists another real number such that the sum of the two real numbers is equal to 1
(b) For every real number there exists another number such that the sum of the two real numbers is equal to 0
(c) The addition of zero to any real number is equal to that real number
(d) For every real number there exists another real number such that the

- product of the two numbers is equal to 1
- (e) The product of any real number with 1 is equal to that real number.

(1 mark)

53. Three simultaneous equations involving three variables can be solved if the three equations $\leq$
Answer
 $\geq$

- (a) Can be satisfied by the three variables assuming equal value in all the equations
- (b) Can be satisfied when two out of the three variables assume equal value in all the equations
- (c) Can be satisfied when each of the three variables assumes a different value in each of the three equations
- (d) Can be satisfied when each of the variables are set to zero in all the equations
- (e) Can be satisfied when the three variables respectively assume the same values in the three equations.

(1 mark)

54. The geometric mean between two given quantities is equal to $\leq$
Answer
 $\geq$

- (a) The sum of the arithmetic mean and harmonic mean between the two quantities
- (b) The difference between the arithmetic mean and harmonic mean between the two quantities
- (c) The geometric mean of the arithmetic mean and the harmonic mean between the two quantities
- (d) The product of the arithmetic mean and the geometric mean between the two quantities
- (e) The ratio of the arithmetic mean to the geometric mean between the two quantities.

(1 mark)

55. Which of the following is **false**? $\leq$
Answer
 $\geq$

- (a) If $\mathbf{b - a > 0}$, then $\mathbf{a < b}$
- (b) If $\mathbf{a < b}$ and $\mathbf{b < c}$, then $\mathbf{a < c}$
- (c) If $\mathbf{a < b}$, then $\mathbf{(a + c) < (b + c)}$
- (d) If $\mathbf{a < b}$ and $\mathbf{x < 0}$, then $\mathbf{ax < bx}$
- (e) If $\mathbf{a < b}$ and $\mathbf{x = 0}$, then $\mathbf{ax = bx}$.

(1 mark)

56. A quadratic equation is of the form $ax^2 + bx + c = 0$, where **a**, **b** and **c** are constants. If **c** is equal to zero, then the roots of the equation are

- (a) Only positive values equal to zero (b) Only negative values (c) All
 (d) $\frac{b}{a}$ or $-\frac{b}{a}$ (e) 0 or $-\frac{b}{a}$.

(1 mark)

57. A straight line with a positive slope

- (a) Rises from left to right as the values increase along the **X**-axis
 (b) Always passes through the point of intersection of the **X** and **Y** axes
 (c) Falls from left to right as the values increase along the **X**-axis
 (d) Is parallel to the **X**-axis
 (e) Is parallel to the **Y**-axis.

(1 mark)

58. Which of the following is **true**?

- (a) In a logarithmic function the logarithmic operation is applied on the dependent variable
 (b) In a logarithmic function the base of the logarithm is always 10
 (c) In a logarithmic function the independent variable cannot be negative
 (d) In a logarithmic function the base of the logarithm can be any real number
 (e) In a logarithmic function the dependent variable cannot be negative.

(1 mark)

59. Which of the following is **false** with regard to the derivative of a function?

- (a) It indicates the rate of change of the function at a given point
 (b) The slope of the tangent to a function at a point is equal to the derivative of the function at the point
 (c) The derivative may be a function of the independent variable
 (d) The derivative of a linear function changes with the value of the independent variable
 (e) If the derivative of a function at a point is negative then it indicates that the function is decreasing at that point.

(1 mark)

60. Which of the following is **true** with regard to the exponential function $y =$

$m \cdot a^x$?

$>$

- (a) The exponential curve falls from left to right as the values increase along the **X**-axis if $m > 0$ and $0 < a < 1$
- (b) The exponential curve falls from left to right as the values increase along the **X**-axis if $m > 0$ and $a > 1$
- (c) The exponential curve is parallel to the **X**-axis if $m > 0$ and $0 < a < 1$
- (d) The exponential curve is parallel to the **X**-axis if $m > 0$ and $a > 1$
- (e) The exponential curve is parallel to the **Y**-axis if $m > 0$ and $a > 1$.

(1 mark)

61. A function $f(x)$ is said to be monotonically increasing if

$\leq$

- (a) The first derivative of $f(x)$ is a constant for all values of x
- (b) The first derivative of $f(x)$ is negative for all values of x
- (c) The first derivative of $f(x)$ is positive for all values of x
- (d) The first derivative of $f(x)$ is zero for all values of x
- (e) The first derivative of $f(x)$ is equal to 1 for all values of x .

[Answer](#)

$\geq$

(1 mark)

62. For a function, $y = f(x)$, every member in the domain is associated with

$\leq$

- (a) Only one member in the range
- (b) Only two members in the range
- (c) Only three members in the range
- (d) Only four members in the range
- (e) Only five members in the range.

[Answer](#)

$\geq$

(1 mark)

63. The limit of a function, $y = f(x)$, at a point $x = a$, is the

$\leq$

- (a) Minimum value of the function for all values of x greater than a
- (b) Minimum value of the function for all values of x less than a
- (c) Maximum value of the function for all values of x greater than a
- (d) Maximum value of the function for all values of x less than a
- (e) Value which the function approaches if x approaches a .

[Answer](#)

$\geq$

(1 mark)

64. Which of the following is **true** with regard to the simplex method of solving linear programming problems?

$\leq$

- (a) This method cannot be applied for solving problems involving two decision variables
- (b) At the optimal solution all the decision variables assume zero values

[Answer](#)

$\geq$

- (c) Slack variables are used to convert 'greater than or equal to' type constraints into equations
- (d) At the optimal solution all the slack variables assume zero values
- (e) The $Z_j - C_j$ value at the bottom of the solution column shows the value of the objective function for any solution.

(1 mark)

65. Which of the following is **not** an assumption underlying linear programming? ≤
[Answer](#)
≥

- (a) The objective to be accomplished can be expressed as a linear function of the decision variables
- (b) The constraints on the use of resources can be expressed as linear equations or inequations
- (c) The amount of resources consumed by each unit of the decision variables is uncertain
- (d) The decision variables can take non-negative values only
- (e) The total consumption of a resource is the sum of the resources consumed by each decision variable.

(1 mark)

66. Which of the following is **true** with regard to the simplex method of solving a linear programming problem (LPP) on profit maximization? ≤
[Answer](#)
≥

- (a) At the optimal solution the slack variables are always equal to zero
- (b) The constraints which contain ' $\leq$ ' are converted into equations by adding slack variables
- (c) There can be only one feasible solution to a LPP
- (d) The slack variables make positive contributions to profit
- (e) The slack variables can only assume the values 0 or 1.

(1 mark)

67. In which of the following conditions, the graphical solution to a linear programming problem will **not** have multiple optimal solutions? ≤
[Answer](#)
≥

- (a) One of the edges of the feasible region coincides with the horizontal axis
- (b) One of the edges of the feasible region is parallel to the horizontal axis
- (c) One of the edges of the feasible region coincides with the vertical axis
- (d) One of the edges of the feasible region is parallel to the vertical axis
- (e) None of the edges of the feasible region is parallel to the objective function

(1 mark)

68. A linear programming problem on profit maximization

- (a) Will have a single optimal solution under all conditions
- (b) Will have an optimal solution which lies outside the feasible region
- (c) Can be solved by the graphical method if there are two decision variables
- (d) Can not be solved by the simplex method if there are two decision variables
- (e) Can be solved for the optimal solution by changing the constraints at every stage.

Answer
≤
≥

(1 mark)

69. Which of the following indicates that the distribution curve of the data is negatively skewed?

- (a) Arithmetic Mean < Median < Mode
- (b) Median < Mode < Arithmetic Mean
- (c) Mode < Arithmetic Mean < Median
- (d) Mode < Median < Arithmetic Mean
- (e) Median < Arithmetic Mean < Mode.

Answer
≤
≥

(1 mark)

70. The sum of the squares of deviations is least when measured from

- (a) Arithmetic mean
- (b) Geometric mean
- (c) Harmonic mean
- (d) Median
- (e) Mode.

Answer
≤
≥

(1 mark)

71. Which of the following is **false** with regard to histograms?

- (a) The widths of the rectangles are proportional to the range of values falling within the classes
- (b) The heights of the rectangles are proportional to the number of items falling within the classes
- (c) The classes are marked along the horizontal axis
- (d) The frequencies are marked along the vertical axis
- (e) The tallest rectangle represents the class which has the least frequency.

Answer
≤
≥

(1 mark)

72. Which of the following statements is **true**?

- (a) The tallest rectangle in a histogram represents the modal class of the

Answer
≤
≥

distribution

- (b) In a symmetrical distribution the mean, median and mode are unequal
- (c) The medians of two sets of data can be combined mathematically
- (d) The median can not be determined graphically
- (e) The mode is always uniquely defined.

(1 mark)

73. Events **A** and **B** are dependent. The joint probability of the events **A** and **B** is

- (a) Equal to the product of the marginal probabilities of the events **A** and **B**
- (b) Not equal to the product of the marginal probabilities of the events **A** and **B**
- (c) Equal to the sum of the marginal probabilities of the events **A** and **B**
- (d) Equal to the difference between the marginal probabilities of the events **A** and **B**
- (e) Is always equal to 1.

(1 mark)

74. If A , B and C are three events such that $P(A)P(B)P(C) > 0$ and $P(B) > P(A)$, which of the following is always **true**?

- (a) $P(\bar{A}) > P(\bar{B})$
- (b) $P(\bar{A}) > P(B)$
- (c) $P(A) > P(\bar{B})$
- (d) $P(\bar{B}) > P(\bar{A})$
- (e) $P(B/C) > P(A/C)$

(1 mark)

75. A die is loaded so that probability of face x is proportional to x . The probability of an odd number occurring when the die is rolled would be

- (a) $\frac{1}{21}$
- (b) $\frac{2}{7}$
- (c) $\frac{3}{7}$
- (d) $\frac{4}{7}$
- (e) $\frac{5}{7}$

(1 mark)

76. Which of the following statements is most appropriate if certain events are mutually exclusive and collectively exhaustive?

- (a) Each of the events has a zero probability
- (b) Some of the events will definitely have zero probability
- (c) The sum of the probabilities of the events will be equal to 1
- (d) The sum of the probabilities of the events will be less than 1
- (e) The sum of the probabilities of the events will be more than 1.

(1 mark)

77. Event **B** is dependent upon event **A**. $P(B) = 0.60$ and $P(A \text{ and } B) = 0.30$, then $P(A/B)$ is ≤
Answer
≥

- (a) 0.18 (b) 0.30 (c) 0.40 (d) 0.50 (e) 0.90.

(1 mark)

78. In a survey of 100 readers, it was found that 50 read magazine **A**, 30 read magazine **B** and 15 read both the magazines. What is the probability of finding a person in the group who reads neither magazine **A** nor magazine **B**? ≤
Answer
≥

- (a) 0.15 (b) 0.35 (c) 0.65 (d) 0.80 (e) 0.85.

(1 mark)

79. An urn contains 3 white and 5 black balls. One ball is drawn from the urn. The probability of that ball being black is ≤
Answer
≥

- (a) $\frac{3}{5}$ (b) $\frac{3}{8}$ (c) $\frac{5}{8}$ (d) $\frac{1}{4}$ (e) $\frac{8}{15}$.

(1 mark)

80. The probabilities of two non-mutually exclusive and independent events, **A** and **B**, are 0.2 and 0.3 respectively. What is the probability of their simultaneous occurrence? ≤
Answer
≥

- (a) 0.01 (b) 0.05 (c) 0.06 (d) 0.15
(e) 0.08

(1 mark)

81. A number is chosen from the first 120 natural numbers. The probability that the number chosen is being a multiple of 5 or 15 is ≤
Answer
≥

- (a) $\frac{1}{5}$ (b) $\frac{1}{8}$ (c) $\frac{1}{16}$ (d) $\frac{1}{9}$ (e) $\frac{1}{10}$.

(1 mark)

82. If the marginal probability of **A** is p_a and that of **B** is p_b , then the joint probability of **A** and **B** will be ≤
Answer
≥

- (a) $p_a + p_b$ (b) $p_a \times p_b$ (c) $p_a + p_b - p_a \times p_b$ (d) $p_a + p_b + p_a \times p_b$
(e) $(1 - p_a)(1 - p_b)$.

(1 mark)

Suggested Answers
Quantitative Methods-I (131) : October 2004

1. Answer : (a)

Reason : In the word "INTEGRATION" number of vowels are : I, E, A, I, O = 5 and number of consonants are : N, T, G, R, T and N = 6.

Keeping five vowels together as one unit, the remaining

six consonants can be arranged in $\frac{6!}{(2!)^2}$ ways and the five vowels can be arranged as $\frac{5!}{2!}$ ways.

$$\text{Total number of ways} = \frac{6!}{(2!)^2} \times \frac{5!}{2!} = 10800.$$

2. Answer : (d)

Reason : We find that the given series is in A.P. with first term as $\log a$ and with a common difference of $\log(a/b)$

Now sum of the n terms of the series is

$$\begin{aligned} S_n &= (n/2) [2 \log a + (n-1) \log(a/b)] \\ &= (n/2) [\log a^2 + n \log(a/b) - \log(a/b)] \\ &= (n/2) [\log ab + n \log(a/b)] \end{aligned}$$

3. Answer : (a)

Reason : Given that $S = (n/2)(a + l)$ or $\frac{2S}{a+l} = n$ (i)

$$l = a + (n-1)d \text{ so that } d = \frac{l-a}{n-1} \text{(ii)}$$

Now putting the value of n from equation (i) in equation (ii), we get

$$d = \frac{(l-a)}{\frac{2S}{(a+l)} - 1} = \frac{(l^2 - a^2)}{2S - (a+l)}.$$

4. Answer : (b)

Reason :

$$\begin{aligned}
 xyz &= \frac{m^{\frac{m-1}{n}}}{n^{\frac{n+1}{m}}} \times \frac{n^{\frac{1-n}{m}}}{m^{\frac{m+3}{n}}} \times \frac{m^{\frac{5}{n}}}{n^{\frac{1-2n}{m}}} \\
 &= \frac{m^{\left(\frac{m-1}{n} - \frac{m+3}{n} + \frac{5}{n}\right)}}{n^{\left(\frac{n+1}{m} - \frac{1-n}{m} + \frac{1-2n}{m}\right)}} \\
 &= \frac{m^{\frac{1}{n}}}{n^{\frac{1}{m}}} = \frac{\sqrt[n]{m}}{\sqrt[m]{n}}
 \end{aligned}$$

5. Answer : (b)

Reason :

$$2x - 3y + 3z = 7 \dots\dots\dots(i)$$

$$y - x + z = 5 \dots\dots\dots(ii)$$

$$2x - 2y + z = 2 \dots\dots\dots(iii)$$

By doing (ii) + 2 × (iii) – (i)

$$x = 5 + 2 \times 2 - 7 = 9 - 7 = 2$$

[TOP](#) $\leq$
 $\geq$

6. Answer : (b)

Reason :

[TOP](#) $\leq$
 $\geq$

Let the numbers be $\frac{m}{r^3}$, $\frac{m}{r}$, mr , mr^3 The product of the four numbers is 4096

i.e. $m^4 = 4096 = 8^4$; So, $m = 8$

The sum of the numbers is $\frac{m}{r^3} + \frac{m}{r} + mr + mr^3 = 85$

$$\text{Or, } 8\left(\frac{1}{r^3} + \frac{1}{r} + r + r^3\right) = 85$$

$$\text{Or, } 8\left(\frac{1}{r^3} + r^3\right) + 8\left(\frac{1}{r} + r\right) = 85$$

Let $\left(\frac{1}{r} + r\right)$ be z

$$\left(\frac{1}{r^3} + r^3\right) = \left(\frac{1}{r} + r\right)^3 - 3\left(\frac{1}{r} + r\right) = z^3 - 3z$$

$$\text{So, } 8(z^3 - 3z) + 8z = 85$$

$$\text{Or, } 8z^3 - 16z = 85$$

$$\text{Or, } (2z - 5)(4z^2 + 10z + 17) = 0$$

The second factor has the imaginary roots so, the feasible root of $z = \frac{5}{2}$

$$\left(\frac{1}{r} + r\right) = \frac{5}{2}$$

$$\text{Or, } 2r^2 + 2 = 5r$$

$$\text{Or, } 2r^2 - 5r + 2 = 0$$

$$\text{Or, } 2r(r - 2) - 1(r - 2) = 0$$

$$\text{Or, } (2r - 1)(r - 2) = 0$$

$$\text{Or, } r = 2 \text{ or } \frac{1}{2}$$

So, the highest number is $8 \times 2^3 = 64$

7. Answer : (c)

Reason :

$${}^{28}C_{2r} : {}^{24}C_{2r-4} = 225 : 11$$

$$\text{Or, } \frac{{}^{28}C_{2r}}{{}^{24}C_{2r-4}} = \frac{225}{11}$$

$$\text{Or, } 11 \times \frac{28!}{2r!(28-2r)!} = 225 \times \frac{24!}{(2r-4)!(28-2r)!}$$

$$\text{Or, } \frac{11 \times 25 \times 26 \times 27 \times 28}{225} = (2r-3)(2r-2)(2r-1)2r$$

$$\text{Or, } (2r-3)(2r-2)(2r-1)2r = \frac{11 \times 25 \times 26 \times 27 \times 28}{225} = \frac{11 \times 26 \times 27 \times 28}{9}$$

$$= 11 \times 26 \times 3 \times 28 = 11 \times 12 \times 13 \times 14$$

$$= 14(14-1)(14-2)(14-3)$$

$$\text{So, } 2r = 14$$

$$\text{Or, } r = 7$$

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8. Answer : (d)

Reason : Total number of possible event = 36.

The set of favorable event is $E = \{x, y\}$, such that $(x + y) \leq 5$,

$E = \{(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (4, 1)\}$

Thus total number of favorable event = 10.

Therefore required probability = $P(E) = \frac{n(E)}{n(S)} = \frac{10}{36} = \frac{5}{18}$

9. Answer : (d)

Reason : The required function,

$$\begin{aligned} f\left(\frac{2x}{1+x^2}\right) &= \log\left(\frac{1+\frac{2x}{1+x^2}}{1-\frac{2x}{1+x^2}}\right) \\ &= \log\left(\frac{1+x^2+2x}{1+x^2-2x}\right) \\ &= \log\left(\frac{1+x}{1-x}\right)^2 = 2\log\left(\frac{1+x}{1-x}\right) = 2f(x) \end{aligned}$$

10. Answer : (b)

Reason : The functions x and y can be rewritten as:

$$\begin{aligned} x &= \frac{1+t}{t^3} = \frac{1}{t^3} + \frac{1}{t^2} \text{ and } y = \frac{3}{2t^2} + \frac{2}{t} \\ \frac{dx}{dt} &= \frac{-3}{t^4} - \frac{2}{t^3} = \frac{-(3+2t)}{t^4}; \\ \frac{dy}{dt} &= \frac{-3}{t^3} - \frac{2}{t^2} = \frac{-(3+2t)}{t^3} \\ \therefore \frac{dy}{dx} &= \frac{dy/dt}{dx/dt} = \frac{-(3+2t)}{t^3} \times \frac{-t^4}{(3+2t)} = t = y' \\ \text{Now } x(y')^3 &= xt^3 = 1+t = 1+y' \end{aligned}$$

11. Answer : (c)

Reason : The give A.P has its first term as, $a = 8$ years and common difference, $d = 1/3$ years

Now given that the sum $S_n = (n/2)[2 \times 8 + (n-1)(1/3)] = 168$

Simplifying the above equation we get $n^2 + 47n - 1008$

$$= 0$$

$$\text{Or, } (n - 16) (n + 63) = 0$$

$\therefore n = 16$. (we ignore the negative value of n).

12. Answer : (e)

Reason : Clearly the numbers between 250 and 1000 which are exactly divisible by 3 are 252, 255, 258, ..., 999.

$$\therefore T_n = 999 = a + (n - 1) d = 252 + (n - 1) 3$$

Solving the above we get $n = 250$.

$$\therefore S = (n/2) [a + l] = (250/2) [252 + 999] = 156375.$$

13. Answer : (a)

Reason : (a, b) implies the set of all real numbers, x , such that, $a < x < b$. Therefore (a, b) excludes all real numbers $x \leq a$ and $x \geq b$.

$[a, b]$ includes all real numbers, x , such that $a \leq x \leq b$.

$(a, b]$ includes all real numbers, x , such that $a < x \leq b$

$[a, b)$ includes all real numbers, x , such that $a \leq x < b$.

14. Answer : (c)

Reason : When two particular persons are together, the remaining 10 soldiers are to be selected from $(n-2)$ in

$${}^{(n-2)}C_{10} \text{ ways. So, } 2m = {}^{(n-2)}C_{10}$$

When three particular persons are together, the remaining 9 soldiers are to be selected from $(n-3)$ in

$${}^{(n-3)}C_9 \text{ ways. So, } m = {}^{(n-3)}C_9$$

$$\text{So, } {}^{(n-2)}C_{10} = 2 \times {}^{(n-3)}C_9$$

$$\text{Or, } \frac{(n-2)!}{10!(n-12)!} = 2 \times \frac{(n-3)!}{9!(n-12)!}$$

$$\text{Or, } n-2 = 2 \times 10 = 20$$

$$\text{Or, } n = 22$$

15. Answer : (d)

Reason :

$$\text{Let } \frac{\log a}{(b-c)} = \frac{\log b}{(c-a)} = \frac{\log c}{(a-b)} = k$$

$$\therefore \log a = k(b-c); \log b = k(c-a); \log c = k(a-b)$$

$$\therefore \log a^a b^b c^c = a \log a + b \log b + c \log c = ak(b-c) + bk(c-a) + ck(a-b) = 0$$

$$\text{So, } a^a b^b c^c = 1$$

16. Answer : (c)

Reason : The given function is

$$\begin{aligned}u &= \frac{x+y}{x-y} \\u_x &= \frac{(x-y)(1) - (x+y)(1)}{(x-y)^2} = \frac{-2y}{(x-y)^2} \\u_y &= \frac{(x-y) - (x+y)(-1)}{(x-y)^2} = \frac{2x}{(x-y)^2} \\\therefore u_x + u_y &= \frac{-2y}{(x-y)^2} + \frac{2x}{(x-y)^2} = \frac{2}{(x-y)}\end{aligned}$$

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17. Answer : (d)

Reason : Given that,

$$\begin{aligned}y^2 &= (x-b)(x-c) \\or, y^2 &= x^2 - (b+c)x + bc \\Differentiating with respect to x, \\2y \frac{dy}{dx} &= 2x - (b+c), \\or, \frac{dy}{dx} &= \frac{2x - (b+c)}{2y} \\Again differentiating w.r.t. x, \\2y \cdot \frac{d^2y}{dx^2} + 2 \frac{dy}{dx} \cdot \frac{dy}{dx} &= 2, \\or, y \cdot \frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 &= 1 \\or, \frac{d^2y}{dx^2} &= \frac{1 - \left(\frac{dy}{dx}\right)^2}{y} = \frac{1 - \left(\frac{2x - (b+c)}{2y}\right)^2}{y} \\At x &= (b+c), \\\frac{d^2y}{dx^2} &= \frac{1 - \left(\frac{b+c}{2\sqrt{bc}}\right)^2}{\sqrt{bc}} = \frac{4bc - b^2 - c^2 - 2bc}{4(bc)^{3/2}} \\&= \frac{-(b^2 + c^2 - 2bc)}{4(bc)^{3/2}} = \frac{-(b-c)^2}{4(bc)^{3/2}}\end{aligned}$$

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18. Answer : (c)

Reason : We have,

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$$\begin{aligned}
 x^{\log_x(x^2-4x+5)} &= (x-1) \\
 \Rightarrow (x^2-4x+5) &= x-1, \left[\because a^{\log_a m} = m \right] \\
 \text{or, } x^2-5x+6 &= 0 \\
 \text{or, } (x-2)(x-3) &= 0 \\
 \Rightarrow x &= 2 \text{ or } 3
 \end{aligned}$$

19. Answer : (e)

Reason : Given that,

$$\begin{aligned}
 e^{f(x)} &= \frac{1+x}{1-x}, \\
 \Rightarrow f(x) &= \log_e \left(\frac{1+x}{1-x} \right)
 \end{aligned}$$

Now finding the value of

$$\begin{aligned}
 f\left(\frac{2x}{1+x^2}\right) &= \log_e \left(\frac{1+\frac{2x}{1+x^2}}{1-\frac{2x}{1+x^2}} \right) \\
 &= \log_e \left(\frac{1+x^2+2x}{1+x^2-2x} \right) = \log_e \left(\frac{1+x}{1-x} \right)^2 \\
 &= 2 \log_e \left(\frac{1+x}{1-x} \right) = 2f(x) \\
 f(x) &= kf\left(\frac{2x}{1+x^2}\right) \\
 f(x) &= k.2f(x) \\
 \therefore k &= \frac{1}{2}
 \end{aligned}$$

20. Answer : (e)

Reason : The given function is,

$$f(x) = \sqrt{\frac{(x+1)(x-2)}{(x+3)(x+6)}}$$

applying log to the base e on both sides, we get

$$\log_e f(x) = \frac{1}{2} [\log_e(1+x) + \log_e(x-2) - \log_e(x+3) - \log_e(x+6)]$$

Differentiating with respect to x ,

$$\begin{aligned}
 \frac{f'(x)}{f(x)} &= \frac{1}{2} \left[\frac{1}{1+x} + \frac{1}{x-2} - \frac{1}{x+3} - \frac{1}{x+6} \right] \\
 \therefore \frac{f'(0)}{f(0)} &= \frac{1}{2} \left[\frac{1}{1} - \frac{1}{2} - \frac{1}{3} - \frac{1}{6} \right] \\
 &= 0.
 \end{aligned}$$

21. Answer : (b)

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Reason : The pth term of the A.P. is $\frac{1}{qr}$ i.e. $A + (p - 1)d = \frac{1}{qr}$

The qth term of the A.P. is $\frac{1}{pr}$ i.e. $A + (q - 1)d = \frac{1}{pr}$

By subtracting the two equations we get

$$(p - q)d = \frac{1}{qr} - \frac{1}{pr} = \frac{(p - q)}{rpq}$$

$$\text{Or, } d = \frac{1}{rpq}$$

The rth term would exceed the pth term by $(r - p)d$. i.e.

$$\frac{(r - p)}{rpq}$$

So the value of rth term is $\frac{1}{rq} + \frac{(r - p)}{rpq} = \frac{1}{rq} + \frac{1}{pq} + \frac{1}{rq} = \frac{1}{pq}$

So the rth term of the H.P. is pq .

22. Answer : (d)

Reason :

$$\log_{12} 27 = a,$$

$$= \frac{\log_x 27}{\log_x (3 \times 4)} = \frac{3 \log_x 3}{\log_x 3 + \log_x 4} = \frac{3 \log_x 3}{\log_x 3 + 2 \log_x 2} = a$$

$$\text{Or, } \frac{\log_x 3 + 2 \log_x 2}{3 \log_x 3} = \frac{1}{a}$$

$$\text{Or, } \frac{1}{3} + \frac{2 \log_x 2}{3 \log_x 3} = \frac{1}{a}$$

$$\text{Or, } \frac{\log_x 2}{\log_x 3} = \frac{3}{2} \left(\frac{1}{a} - \frac{1}{3} \right) = \frac{3-a}{2a} \dots\dots\dots(i)$$

$$\text{Now, } \log_6 16 = \frac{\log_x 16}{\log_x 6} = \frac{4 \log_x 2}{\log_x 2 + \log_x 3} = \frac{4 \frac{3-a}{2a} \log_x 3}{\frac{3-a}{2a} \log_x 3 + \log_x 3} = \frac{4(3-a)/2a}{\frac{(3-a)+2a}{2a}}$$

$$= \frac{4(3-a)}{(3-a)+2a} = \frac{4(3-a)}{(3+a)}$$

23. Answer : (a)

Reason :

$$\lim_{t \rightarrow 0} \frac{2 - \sqrt{4 - t^2}}{t^2} = \lim_{t \rightarrow 0} \frac{(2 + \sqrt{4 - t^2})(2 - \sqrt{4 - t^2})}{t^2 (2 + \sqrt{4 - t^2})}$$

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$$\begin{aligned}
&= \lim_{t \rightarrow 0} \frac{4 - (4 - t^2)}{t^2 \left(2 + \sqrt{4 - t^2} \right)} \\
&= \lim_{t \rightarrow 0} \frac{t^2}{t^2 \left(2 + \sqrt{4 - t^2} \right)} \\
&= \lim_{t \rightarrow 0} \frac{1}{2 + \sqrt{4 - t^2}} \\
&= \frac{1}{2 + 2} = \frac{1}{4}
\end{aligned}$$

24. Answer : (c)

Reason :

The function is valid for all positive values of

$$\log \left[(5x - x^2) / 6 \right] \text{ i.e } \log \left[(5x - x^2) / 6 \right] \geq 0 .$$

$$\text{Or, } (5x - x^2) / 6 \geq 1$$

$$\text{Or, } 5x - x^2 \geq 6$$

$$\text{Or, } x^2 - 5x \leq 6$$

$$\text{Or, } (x - 2)(x - 3) \leq 0$$

So, the domain is $2 \leq x \leq 3$ or, $[2, 3]$

25. Answer : (c)

Reason :

$$\text{Let } a^{bx-c} = p \text{ Or, } bx-c = \log_a p = \frac{\log p}{\log a}$$

Now differentiating both sides with respect to x

$$\frac{d(bx-c)}{dx} = \frac{1}{\log a} \times \frac{d \log p}{dx} = \frac{1}{\log a} \times \frac{d \log p}{dp} \times \frac{dp}{dx}$$

$$\text{Or, } b = \frac{1}{\log a} \times \frac{1}{p} \times \frac{dp}{dx}$$

$$\text{Or, } \frac{dp}{dx} = bp \times \log a$$

Now by putting $p = a^{bx-c}$

$$\text{Or, } \frac{da^{bx-c}}{dx} = ba^{bx-c} \times \log a$$

26. Answer : (a)

Reason :

$$\frac{d(\sqrt{x} + \sqrt{y})}{dx} = 0$$

$$\text{Or, } \frac{d(\sqrt{x})}{dx} + \frac{d(\sqrt{y})}{dx} = 0$$

$$\text{Or, } \frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}} \frac{dy}{dx} = 0$$

$$\text{Or, } \frac{dy}{dx} = -\frac{\sqrt{y}}{\sqrt{x}} = -\sqrt{\frac{y}{x}}$$

27. Answer : (e)

Reason : $\int_0^1 2x e^{x^2+5} dx = \left[e^{x^2+5} \right]_0^1 = [e^6 - e^5] = e^5(e - 1)$

Since, $\frac{d}{dx}(e^{f(x)}) = f'(x) \cdot e^{f(x)}$

$\therefore \frac{d}{dx}(e^{x^2+5}) = (2x + 0)e^{x^2+5} = 2xe^{x^2+5}$

28. Answer : (d)

Reason : The function satisfies $f(2) > f(0)$

29. Answer : (b)

Reason : $f(x) = 50x^2 - 400x + 1000$

$f'(x) = 50(2x) - 400 = 100x - 400$

At minima, $f'(x) = 0$ and $f''(x) > 0$

$f'(x) = 0 \Rightarrow 100x - 400 = 0 \Rightarrow x = \frac{400}{100} = 4$

$f''(x) = 100 > 0$

$\therefore f(x)$ is minimum at $x = 4$.

30. Answer : (d)

Reason : Total cost is given by: $T(x) = 10500x - 20x^2 + \frac{1}{5}x^3$

Average cost = $\frac{\text{Total cost}}{\text{Number of units}}$

$\therefore \text{Average cost, } U(x) = \frac{1}{x} \left[10500x - 20x^2 + \frac{1}{5}x^3 \right]$

or $U(x) = 10500 - 20x + \frac{1}{5}x^2$

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$$U'(x) = -20 + \frac{2}{5}x$$

For maxima or minima $U'(x) = 0$

$$U'(x) = 0 \Rightarrow -20 + \frac{2}{5}x = 0 \text{ or } x = \frac{20 \times 5}{2} = 50$$

$$U''(x) = \frac{2}{5} > 0 \text{ always}$$

$\therefore$ The average cost of the product is minimum when number of units produced is 50.

31. Answer : (d)

Reason : the demand and cost functions are $P = 1200 - 6Q$, $C = Q^3 - 6Q^2 + 120Q + 600$.

$$\text{Revenue} = P \times Q = (1200 - 6Q) \times Q = 1200Q - 6Q^2.$$

$$\text{Marginal revenue} = dR/dQ = 1200 - 12Q$$

$$\text{Marginal cost} = dC/dQ = 3Q^2 - 12Q + 120$$

Marginal cost and marginal revenue at an output of 15 units is

$$MR = 1200 - 12 \times 15 = 1020$$

$$MC = 3 \times 225 - 12 \times 15 + 120 = 675 - 180 + 120 = 615.$$

32. Answer : (b)

Reason : At the point B, both the first derivative and the second derivative of the function are decreasing.

33. Answer : (a)

Reason : If the program is formulated we get it as follows:

Objective Function: Max: $100X_1 + 125X_2$

Constraints: $X_1 + 2X_2 \leq 2000$ So, at the most

$$X_1 + 2X_2 = 2000 \dots\dots\dots(i)$$

$X_1 + 3X_2 \leq 2700$ So, at the most

$$X_1 + 3X_2 = 2700 \dots\dots\dots(ii)$$

$X_1 + X_2 \leq 1400$ So, at the most $X_1 + X_2 = 1400$

$\dots\dots\dots(iii)$

At $X_1 = 0$; values of X_2 from different equations would be 1000, 900, 1400. So, the vertex of the feasible region along the X_2 axis would be (0, 900).

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At $X_2 = 0$; values of X_1 from different equations would be 2000, 2700, 1400. So, the vertex of the feasible region along the X_1 axis would be (1400, 0).

The point of intersection between eqn(i) and (ii) would be:

By (ii) – (i) –

$$X_2 = 2700 - 2000 = 700.$$

Putting the value of X_2 in equation (i) –

$$X_1 = 2000 - 1400 = 600.$$

So, the point is (600, 700) . The point is satisfying the inequation (ii) as $600 + 700 \leq 1400$. The value of total contribution at this point is $600 \times 100 + 700 \times 125 = \text{Rs. } 1,47,500$

The point of intersection between eqn(i) and (iii) would be:

By (i) – (iii) –

$$X_2 = 2000 - 1400 = 600.$$

Putting the value of X_2 in equation (i) –

$$X_1 = 2000 - 1200 = 800.$$

So, the point is (800, 600) . The point is satisfying the inequation (iii) as $800 + 3 \times 600 \leq 2700$. The value of total contribution at this point is $800 \times 100 + 600 \times 125 = \text{Rs. } 1,55,000$

The point of intersection between eqn(ii) and (iii) would be:

By (ii) – (iii) –

$$2X_2 = 2700 - 1400 = 1300.$$

$$\text{Or, } X_2 = 650$$

Putting the value of X_2 in equation (ii) –

$$X_1 = 2700 - 1950 = 750.$$

So, the point is (750, 650) . The point is not satisfying the inequation (i) as $750 + 2 \times 650 \geq 2000$. The point is not in the feasible region.

So, the optimum point is where the contribution is maximum i.e. (800, 600)

34. Answer : (d)

Reason : The objective function of the problem is

$$\text{Maximize: } 1125X_1 + 1650X_2$$

The constraints of the problem are as follows:

Capital budget:

$$10X_1 + 15X_2 \leq 300$$

$$\text{Or, } 2X_1 + 3X_2 \leq 60$$

So the terminal value of the equation is $2X_1 + 3X_2 = 60 \dots(i)$

Operating Budget:

$X_1 + X_2 \leq 25$ So, the terminal value of the equation is $X_1 + X_2 = 25 \dots(ii)$

Planning for increased future demand:

$X_1 \leq X_2$. So, the terminal value of the equation is $X_1 - X_2 = 0 \dots(iii)$

Solving (i) and (ii) we get $X_1 = 15$ and $X_2 = 10$. But this does not satisfy the third constraint, so, this vertex is not in the feasible region.

Solving (ii) and (iii) we get $X_1 = 12.5$ and $X_2 = 12.5$. But this does not satisfy the first constraint as $2 \times 12.5 + 3 \times 12.5 = 62.5 \geq 60$. So, this point is also not in the feasible region.

Solving (i) and (iii) we get $X_1 = 12$ and $X_2 = 12$. This point also satisfies the constraint (ii) as $12 + 12 = 24 \leq 25$.

From (iii) we get that there cannot be any point on X-axis i.e. keeping $X_2 = 0$. Equation (i) and (ii) cross the y-axis at (0, 20) and (0, 25) respectively. So the point in the feasibility region would be (0, 20).

Now, between (0, 20) and (12, 12), whichever would maximize the objective function would be the optimal solution.

Additional capacity at (12, 12) :

$$1125 \times 12 + 1650 \times 12 = 33300 \text{ Passenger-Kilometers per hour}$$

Additional capacity at (0, 20) :

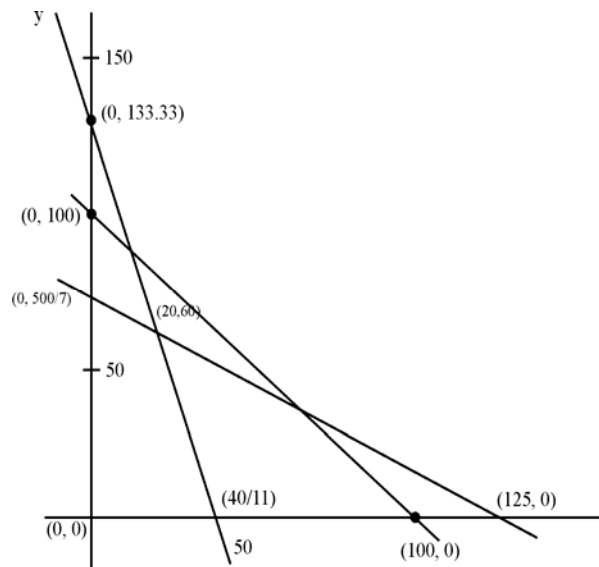
$$1125 \times 0 + 1650 \times 20 = 33000 \text{ Passenger-Kilometers per hour}$$

So, the optimal solution is (12, 12), that means value of $X_2 = 12$.

35. Answer : (c)

Reason : Consider the corresponding system of linear equations and the feasible region:

$$\begin{cases} x + y = 100 \\ 120x + 210y = 15000 \\ 110x + 30y = 4000 \\ x, y = 0 \end{cases}$$



Check the vertices to find that the maximum value is **6460 at (20,60)**

36. Answer : (c)

Reason : Total number of days available to Zaheda is 31. She takes one full day to sew 4 blue squares or 7 white squares. Now if she is sewing b and w squares respectively total days consumed would be $\frac{b}{4} + \frac{w}{7}$ which cannot be more than 31 days. So, the constraint (c) is the answer.

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37. Answer : (a)

Reason : The variance of the population using assumed mean can be calculated as:

$$\sigma^2 = \left[\frac{\sum fd^2}{N} - \left(\frac{\sum fd}{N} \right)^2 \right] \times i^2$$

Deviations	d = $\frac{\text{Deviation}}{i}$	Frequency	d ²	fd ²	Fd
-20	-2	5	4	20	-10
-10	-1	10	1	10	-10
0	0	20	0	0	0
10	1	30	1	30	30
20	2	20	4	80	40
30	3	15	9	135	45
		100		275	95

$$\begin{aligned} \text{Variance of the population} &= \left[\frac{\sum fd^2}{N} - \left(\frac{\sum fd}{N} \right)^2 \right] i^2 \\ &= \left[\frac{275}{100} - \left(\frac{95}{100} \right)^2 \right] \times (10)^2 \\ &= 184.75. \end{aligned}$$

38. Answer : (c)

Reason : Since the difference distances have been covered at different speeds, the weighted harmonic mean may be calculated as follows:

Speed (v) km/ hr	Distance (s) (Kms)	s/v
30	25	0.83
40	50	1.25
100	10	0.10
24	24	1.00
Total	109	3.18

The average speed may be calculated as:

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$$\frac{\sum S_i}{\sum \frac{S_i}{v_i}} = \frac{109}{3.18} = 34.28 \text{ km per hour}$$

39. Answer : (c)

Reason : Average time taken now = $\frac{(50 \times 4) + (125 \times 5) + (75 \times 6)}{(50 + 125 + 75)} = 5.1$ hours

Average time taken after three years = $\frac{4(50 + 125) + (75 \times 5)}{(50 + 125) + 75} = 4.3$ hours

Change in time taken = $5.1 - 4.3 = 0.80$ hours reduction.

40. Answer : (a)

Reason : From the given values, $f_1 + f_2 = 229 - (12 + 30 + 65 + 25 + 18) = 79$. Since, median is given as 46, the class 40 – 60 may be termed as the median class. Therefore, using median formulae, we get,

$$46 = 40 + \frac{115 - (12 + 30 + f_1 + 1)}{65} \times 10 \quad \text{or, } 6 = \frac{72 - f_1}{6.5} \quad \text{or, } f_1 = 72 - 39 = 33 \text{ while } f_2 = 79 - 33 = 46.$$

41. Answer : (e)

Reason : Total number of bulbs = 20

Number of defectives = 5

Number of non-defectives = $20 - 5 = 15$

$$P(\text{all three bulbs are defective}) = \frac{{}^5C_3}{{}^{20}C_3} = \frac{10}{1140} = \frac{1}{114}$$

P(exactly one of the three bulbs is defective) =

$$\frac{{}^5C_1 \times {}^{15}C_2}{{}^{20}C_3} = \frac{5 \times 105}{1140} = \frac{35}{76}$$

P(at least one of the three bulbs is defective) = $1 - P(\text{none of the bulbs is defective})$

$$= 1 - \frac{{}^{15}C_3}{{}^{20}C_3} = 1 - \frac{455}{1140} = \frac{137}{228}$$

P(at least two of the three bulbs are not defective) = P(2 are not defective) + P(3 are not defective)

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$$= \frac{{}^{15}C_2 \times {}^5C_1}{{}^{20}C_3} + \frac{{}^{15}C_3}{{}^{20}C_3} = \frac{5 \times 105}{1140} + \frac{455}{1140} = \frac{49}{57}$$

(e) P(at least one of the three bulbs is not defective)

= 1 – P (All the three bulbs are defective)

$$= 1 - \frac{{}^5C_3}{{}^{20}C_3} = 1 - \frac{10}{1140} = \frac{113}{114}$$

42. Answer : (d)

Reason : Let the following notations be used,

A : A is appointed as CEO.

B : B is appointed as CEO.

C : C is appointed as CEO.

N : New product is launched.

Given: P (A) = 0.20

P (B) = 0.30

P (C) = 0.50

P (N/A) = 30% = 0.30

P (N/B) = 50% = 0.50

P (N/C) = 60% = 0.60

∴ P (N) = P (A and N) + P (B and N) + P (C and N)

= (0.20 × 0.30) + (0.30 × 0.50) + (0.50 × 0.60) = 0.51 i.e. 51%.

43. Answer : (c)

Reason : We are given that P(G₁) = 0.5, P(G₂) = 0.6 and P(G₃) = 0.8 and hence their complementary events are given by:

$P(\bar{G}_1) = 0.5$, $P(\bar{G}_2) = 0.4$ and $P(\bar{G}_3) = 0.2$.

The event of exactly one hit may be the combinations of the following mutually exclusive events:

(i) G_1 and $\bar{G}_2$ and $\bar{G}_3$ happens (ii) $\bar{G}_1$ and G_2 and $\bar{G}_3$ happens
or

(iii) $\bar{G}_1$ and $\bar{G}_2$ and G_3 happens

Therefore from the additional theory of probability, the required probability may be obtained as:

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$$\begin{aligned}
P &= P(G_1 \text{ and } \bar{G}_2 \text{ and } \bar{G}_3) + P(\bar{G}_1 \text{ and } G_2 \text{ and } \bar{G}_3) + P(\bar{G}_1 \text{ and } \bar{G}_2 \text{ and } G_3) \\
&= P(G_1)P(\bar{G}_2)P(\bar{G}_3) + P(\bar{G}_1)P(G_2)P(\bar{G}_3) + P(\bar{G}_1)P(\bar{G}_2)P(G_3) \\
&= 0.5 \times 0.4 \times 0.2 + 0.5 \times 0.6 \times 0.2 + 0.5 \times 0.4 \times 0.8 = \\
&0.04 + 0.06 + 0.16 = 0.26
\end{aligned}$$

44. Answer : (c)

Reason : The probability that both S and T occurs is given by $P(ST)$, the occurrence of S but not T is given by $P(S\bar{T})$ while the occurrence of T but not S is given by $P(\bar{S}T)$. Now, as per the terms of the problem, $P(ST) = P(S\bar{T}) = P(\bar{S}T) = p$

Therefore, $P(ST) + P(S\bar{T}) + P(\bar{S}T) = 3p$ or, $P(ST) + P(S) - P(ST) + P(T) - P(ST) = 3p$
or, $P(S) + P(T) - P(ST) = 3p$ or, $P(S + T) = 3p$

45. Answer : (c)

Reason : According to Baye's theorem:

$$\begin{aligned}
P(\text{Female/Illiterate}) &= \frac{P(\text{Female and Illiterate})}{P(\text{Illiterate})} \\
P(\text{Female and Illiterate}) &= P(\text{Female}) \cdot P(\text{Illiterate} / \text{Female})
\end{aligned}$$

$$\begin{aligned}
&= \left(1 - \frac{3}{5}\right) \times \frac{4}{5} = \frac{2}{5} \times \frac{4}{5} = \frac{8}{25} \\
\frac{P(\text{Illiterate})}{P(\text{Male and Illiterate})} &= \frac{P(\text{Female and Illiterate})}{P(\text{Male and Illiterate})} +
\end{aligned}$$

$$\begin{aligned}
&\frac{8}{25} + [P(\text{Male}) \cdot P(\text{Illiterate} / \text{Male})] \\
&= \frac{8}{25} + \left[\frac{3}{5} \times \frac{1}{2}\right] = \frac{8}{25} + \frac{3}{10} = \frac{31}{50}
\end{aligned}$$

$$\begin{aligned}
&\frac{\frac{8}{25}}{\frac{31}{50}} = \frac{16}{31} \\
\therefore P(\text{Female} / \text{Illiterate}) &= \frac{16}{31} = 0.516 \text{ i.e. } 51.6\% \text{ approximately.}
\end{aligned}$$

(In the above calculations, 'males' imply adult males and 'females' imply adult females).

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46. Answer : (e)

Reason : $P(\bar{A}B) + P(A\bar{B})$ indicates the occurrence of at least any one of the two events. The first component indicates the probability of the occurrence of B but not A while the second component indicates the occurrence of A not B. Hence, the option (e) is happened to be the right choice.

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47. Answer : (c)

Reason : (a) The classical approach to probability assumes that the outcomes are equally likely.

(b) In the relative frequency approach to probability the probability of an event is determined after performing the experiment large number times.

(c) In the classical approach to probability the probability of an event is determined before performing the experiment.

(d) The classical approach to probability assumes that all possible outcomes of the experiment are known.

(e) The classical approach can be used to find out the probability of mutually exclusive events.

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48. Answer : (e)

Reason : If B, C and D are mutually exclusive and collectively exhaustive, and A is another event which can jointly occur with B, C or D, then

$$P(A) = P(A \text{ and } B) + P(A \text{ and } C) + P(A \text{ and } D).$$

This means that $P(A)$ is equal to the sum of all the joint probabilities which include event A.

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49. Answer : (b)

Reason : Out of 200 numbers, one may be chosen by 200 ways. The number of integer that are divisible by 6 are 33 while the number of integers that are divisible by 8 are 25. Now, the lowest common multiple of 6 and 8 is 24 and the number of integer that are divisible by 24 are 8.

Hence, $P(A)$ i.e. the probability that a number is divisible by 6 is $33/200$ while the $P(B)$ i.e. the probability that a number is divisible by 8 is $25/200$

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while $P(AB)$ i.e. a number that is divisible by both 6 and 8 is $8/200$. Hence, the required probability is given by
 $P(A \text{ or } B) = P(A) + P(B) - P(AB) = 33/200 + 25/200 - 8/200 = 50/200 = 1/4 = 0.25$.

50. Answer : (b)

Reason : Let the following notations be used:

A : Pipe is produced by Plant A.

B : Pipe produced by Plant B.

C : Pipe is produced by Plant C.

D : Pipe is defective.

$$\text{Given: } P(A) = \frac{1000}{1000+2000+4000} = \frac{1000}{7000} = \frac{1}{7}$$

$$P(B) = \frac{2000}{1000+2000+4000} = \frac{2000}{7000} = \frac{2}{7}$$

$$P(C) = \frac{4000}{1000+2000+4000} = \frac{4000}{7000} = \frac{4}{7}$$

$$P(D/A) = 2\% = 0.02$$

$$P(D/B) = 3\% = 0.03$$

$$P(D/C) = 5\% = 0.05$$

$$P(A \text{ and } D) = P(A) \cdot P(D/A) =$$

$$\frac{1}{7} \times 0.02 = \frac{0.02}{7}$$

$$P(B \text{ and } D) = P(B) \cdot P(D/B) =$$

$$\frac{2}{7} \times 0.03 = \frac{0.06}{7}$$

$$P(C \text{ and } D) = P(C) \cdot P(D/C) =$$

$$\frac{4}{7} \times 0.05 = \frac{0.20}{7}$$

$$\therefore P(D) = P(A \text{ and } D) + P(B \text{ and } D) + P(C$$

$$\text{and } D) = \frac{0.28}{7}$$

$$P(B/D) = \frac{P(B \text{ and } D)}{P(D)} = \frac{(0.06/7)}{(0.28/7)} = 0.2143. \text{ i.e.}$$

21.43%

51. Answer : (b)

Reason :

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$$\left(\frac{b}{a} - \frac{c}{b}\right) = \frac{b^2 - ac}{ab}$$

As, $a < c < b$, and b is negative so a and c would also be negative. Highest negative number would always have lowest absolute number. So $b^2 < ac$. Moreover as both a and b are negative numbers so ab is a positive number. On the whole it would be a negative number i.e. less than zero.

52. Answer : (b)

Reason : a. This is not a property of the real numbers

b. According to the inverse property of addition for every real number there exists another number such that the sum of the two real numbers is equal to 0.

c. This is the identity property of addition.

d. This is the inverse property of multiplication.

e. This is the identity property of multiplication.

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53. Answer : (e)

Reason : Three simultaneous equations involving three variables can be solved if the three equations can be satisfied when the three variables respectively assume the same values in each of the three equations.

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54. Answer : (c)

Reason : (c) The geometric mean between two given quantities is equal to the geometric mean of the arithmetic mean and the harmonic mean between the two given quantities.

(a), (b), (d) and (e) are all incorrect with regard to the geometric mean between two given quantities.

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55. Answer : (d)

Reason : If $a < b$ and $x < 0$ then $ax > bx$. All other statements are true.

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56. Answer : (e)

Reason : $ax^2 + bx + c = 0$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

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$$\therefore \text{ For } c = 0 \quad x = \frac{-b \pm \sqrt{b^2 - 0}}{2a} = \frac{-b \pm b}{2a} = 0 \text{ or } -\frac{b}{a}$$

57. Answer : (a)

Reason : A straight line with a positive slope rises from left to right as the values increase along the X-axis because with each higher value of 'x' the product of the slope and the value of 'x' increases with the intercept remaining constant. Hence (a) is true. (b) is not true because the straight line passes through the origin only if the intercept is zero and it can happen both in the cases of a positive slope as well as a negative slope. (c) is not true because the straight line falls from left to right as the values along X-axis increase only when the slope is negative. (d) will be true only if 'y' has a constant value. (e) will be true only if 'x' has a constant value.

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58. Answer : (c)

Reason : a. In a logarithmic function the logarithmic operation is applied on the independent variable.
b. In a logarithmic function the base of the logarithm may be any number other than 10.
c. In a logarithmic function the independent variable cannot be negative.
d. In a logarithmic function the base of the logarithm cannot be zero or negative numbers.

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59. Answer : (d)

Reason : (a) The derivative of a function indicates the rate of change of the function.

The slope of the tangent to a function at a point is equal to the derivative of the function at that point.

(c) The derivative of a function can be said to be a function of the independent variable if the expression of the derivative contains the independent variable.

(d) The derivative of a linear function is the slope of the linear function, which is a constant value for all values of the dependent variable.

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(e) If the derivative of any function at a point is negative then it indicates that the function is decreasing at that point.

60. Answer : (a)

Reason : The exponential function falls from left to right as the values increase along the X-axis if $m > 0$ and $0 < a < 1$. Hence (a) is true. (b), (c) (d) and (e) are not true because they are not the characteristics of the exponential function.

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61. Answer : (c)

Reason : a. If the first derivative of **f(x)** is a constant for all values of **x** then it indicates that the function changes at a constant rate.

b. A function **f(x)** is said to be monotonically decreasing if the first derivative of **f(x)** is negative for all values of **x**.

c. A function **f(x)** is said to be monotonically increasing if the first derivative of **f(x)** is positive for all values of **x**.

d. If the first derivative of **f(x)** is zero for all values of **x** then it indicates that the function has a constant value.

e. This is not the condition for a monotonically increasing function.

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62. Answer : (a)

Reason : For a function $y = f(x)$, every member in the domain is associated with only one member in the range.

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63. Answer : (e)

Reason : The limiting value of a function, $y = f(x)$ at $x = a$ is defined as the value to which **y** approaches, as **x** approaches **a**; it is neither the minimum value, nor the maximum value.

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64. Answer : (e)

Reason : The following are **true** with regard to simplex method of solving linear programming problems:

a. This method can be applied for solving problems

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involving two decision variables.

- b. At the optimal solution all the decision variables assume non-negative values.
- c. Slack variables are used convert 'less than or equal to' type constraints into equations.
- d. At the optimal solution all the slack variables assume non-negative values.
- e. The $Z_j - C_j$ value at the bottom of the solution column shows the value of the objective function for any solution.

65. Answer : (c)

Reason : (a), (b), (d) and (e) are all assumptions underlying linear programming. However, (c) is not an assumption because linear programming assumes that the amount of resources consumed by each unit of the decision variables is certain.

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66. Answer : (b)

Reason : (a) At the optimal solution the slack variables need not be equal to zero.

- (b) The constraints which contain the $\leq$ sign are converted into equations by adding slack variables.
- (c) There can be more than one feasible solution to a LPP.
- (d) The slack variables make zero contribution towards profit.
- (e) The slack variables can only assume any non-negative value.

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67. Answer : (e)

Reason : A graphical solution to a linear programming problem will have multiple optimal solutions if the objective function is parallel to an edge of the feasible region which is in the direction of the optimal movement of the objective function. If none of the edges of the feasible region is parallel to the objective function then, the possibility of multiple optimal solution does not arise.

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68. Answer : (c)

Reason : A linear programming problem (LPP) on profit maximization may have multiple optimal solutions. The optimal solution cannot lie outside the feasible region. A LPP can be solved by the simplex method when it contains two decision variables. A LPP cannot be solved by changing the constraints at every stage, because changing the constraints changes the definition of the problem. A LPP can be solved by the graphical method if contains two decision variables. Hence (a), (b), (d) and (e) are not correct; (c) is correct.

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69. Answer : (a)

Reason : In case of a negatively skewed distribution arithmetic mean < median < mode. Hence (a) is the answer. The relationships stated in (b), (c), (d) and (e) are not the features of negatively skewed distributions.

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70. Answer : (a)

Reason : The sum of the squares of deviations is least when measured from arithmetic mean in stead of taking from any other arbitrarily chosen value.

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71. Answer : (e)

Reason : (a) The widths of the rectangles are proportional to the range of values falling within the classes.
(b) The heights of the rectangles are proportional to the number of values falling within the classes.
(c) The classes are marked along the horizontal axis.
(d) The frequencies are marked along the vertical axis.
(e) The tallest rectangle represents the class which has the maximum frequency.

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72. Answer : (a)

Reason : (a) The tallest rectangle in a histogram represents the modal class.
(b) In a symmetrical distribution mean, median and mode are equal.
(c) The median can not be mathematically manipulated; hence medians of two sets of data can not be

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combined.

(d) The median can be determined graphically.

(e) The mode is not uniquely determined when more than one observations have the highest frequency.

73. Answer : (b)

Reason : (a) & (b) For two dependent events **A** and **B**, the joint probability of the events **A** and **B** is not equal to the product of their marginal probabilities.

For two dependent events **A** and **B**, the joint probability of the events **A** and **B** is not equal to the sum of their marginal probabilities.

For two dependent events **A** and **B**, the joint probability of the events **A** and **B** is not equal to the difference between their marginal probabilities.

For two dependent events **A** and **B**, the joint probability of the events **A** and **B** is not always equal to 1.

74. Answer : (a)

Reason : If $P(B) > P(A)$ then $-P(A) > -P(B)$ i.e. $1 - P(A) > 1 - P(B)$ i.e. $P(\bar{A}) > P(\bar{B})$ Hence, the answer.

75. Answer : (c)

Reason : There are six faces in a die and they are named as 1, 2, 3, 4, 5 and 6. So, the corresponding probability would be $1k, 2k, 3k, 4k, 5k$ and $6k$ respectively. Since one of the face must appear the sum of all these probabilities should be one i.e. $k+2k+3k+4k+5k+6k = 21k = 1$ or $k = \frac{1}{21}$

The probability of odd number is $1k+3k+5k = 9k = \frac{9}{21}$
 $= \frac{3}{7}$.

76. Answer : (c)

Reason : If certain events are mutually exclusive and collectively exhaustive then only one of them can happen at a time and the entire sample space is divided among the two events. Hence the sum of their

probabilities will be equal to 1.00.

77. Answer : (d)

Reason : $P(A \text{ and } B) = P(B) \times P(A/B)$

Or $0.30 = 0.60 \times P(A/B)$

Or $P(A/B) = \frac{0.30}{0.60} = 0.50$

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78. Answer : (b)

Reason : $P(A) = \frac{50}{100} = 0.50$ $P(B) = \frac{30}{100} = 0.30$

$P(A \text{ and } B) = \frac{15}{100} = 0.15$

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$
 $= 0.50 + 0.30 - 0.15 = 0.65$

$\therefore P(\text{Neither } A \text{ nor } B) = 1 - P(A \text{ or } B) = 1 - 0.65 = 0.35.$

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79. Answer : (c)

Reason : There are total eight balls in the urn. From there one ball can be drawn in total n ways.

So $n = {}^8C_1 = 8.$

There are five black balls, the number of ways one black ball can drawn $m = {}^5C_1 = 5.$

So, the probability is $\frac{m}{n} = \frac{5}{8}.$

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80. Answer : (c)

Reason : For non-mutually exclusive and independent events A and B,

$P(A \text{ and } B) = P(A) \cdot P(B)$

$P(A) = 0.2$

$P(B) = 0.3$

$P(A \text{ and } B) = 0.2 \times 0.3 = 0.06$

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81. Answer : (a)

Reason : A number may be chosen from the set of first 120 natural numbers by 120 ways. Now, the number of numbers that are multiple of 5 or 15 is 24. Hence, the

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required probability is $24/120 = 1/5$.

82. Answer : (b)

Reason : The joint probability of any two events is estimated by multiplying the probabilities of the respective events.

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